Math 10B with Professor Stankova
Quiz 7; Tuesday, 3/12/2019
Section #206; Time: 9:30 AM
GSI name: Roy Zhao **Name:** ______________________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. True **FALSE** The geometric distribution assumes that trials are dependent (without replacement) while the binomial distribution assumes that trials are independent.

   Solution: The geometric and binomial distribution both assume that the trials are independent.

2. **TRUE** False If f is the PMF of a random variable X, it is possible for $f(E[X]) = 0$.

   Solution: An example is a die roll. The expected value of a die roll is 3.5 but it is not possible to roll a 3.5.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (3 points) At the Olympics, suppose that the number of medals per athlete is Poisson distributed with an average of 0.5 medals per athlete. What is the probability that in a team of 12 athletes, they have 6 medals amongst them?

   Solution: In a team of 12, we expect to see $\lambda = 12 \cdot 0.5 = 6$ medals amongst them. This is Poisson distributed so the probability of them having 6 medals among them is $f(6) = \frac{\lambda^6 e^{-\lambda}}{6!} = \frac{6^6 e^{-6}}{6!}$.

(b) (4 points) Now suppose I go to a party with 40 random athletes at the Olympic Village and on average, I expect to see 10 gold medalists there. If there are 100 gold medalists total at the Olympics, how many athletes are at the Olympics?

Solution: This is a hyper-geometric distribution because out of the total N athletes, there are $m = 100$ gold medalist and at a party of $n = 40$ people, we expect to see $E[X] = 10$ of them. Thus, we have that

$$10 = E[X] = \frac{mn}{N} = \frac{100 \cdot 40}{N}.$$

So $N = \frac{100 \cdot 40}{10} = 400$.

(c) (3 points) With the same numbers as part (b), suppose that I throw a party and invite 10 random athletes. What is the probability that amongst them, 4 of them are gold medalists?

Solution: As said before, this is a hyper-geometric distribution with $m = 100$ and $N = 400$. Then $n = 10$ because there are 10 athletes picked and we want to calculate the probability that we have $k = 4$ gold medalists. The probability of this is

$$f(4) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}} = \frac{\binom{100}{4}\binom{300}{6}}{\binom{400}{10}}.$$